Exercise 11

Verify that $u(x,y) = f(x)g(y)$ is a solution of the PDE $uu_{xy} = u_x u_y$ for all pairs of (differentiable) functions f and g of one variable.

Solution

Suppose that f and g are differentiable functions of x and y, respectively. Then

$$\begin{aligned} u_x &= f'(x)g(y) \\ u_y &= f(x)g'(y) \\ u_{xy} &= f'(x)g'(y) \end{aligned}$$

Substituting the expressions,

$$\begin{aligned} uu_{xy} &= [f(x)g(y)][f'(x)g'(y)] \\ &= f(x)g(y)f'(x)g'(y) \\ &= f'(x)g(y)f(x)g'(y) \\ &= [f'(x)g(y)][f(x)g'(y)] \\ &= u_x u_y \end{aligned}$$

Therefore, $u(x,y) = f(x)g(y)$ is a solution of the PDE, provided that the functions f and g are differentiable.